Charmonia and bottomonia in hot medium and heavy quark diffusion from lattice QCD at finite temperature

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in collaboration with

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BROOKHAVEN NATIONAL LABORATORY

New Progress in Heavy Ion Collisions: What is Hot in the QGP

CCNU, Wuhan, China

Oct. 7, 2015

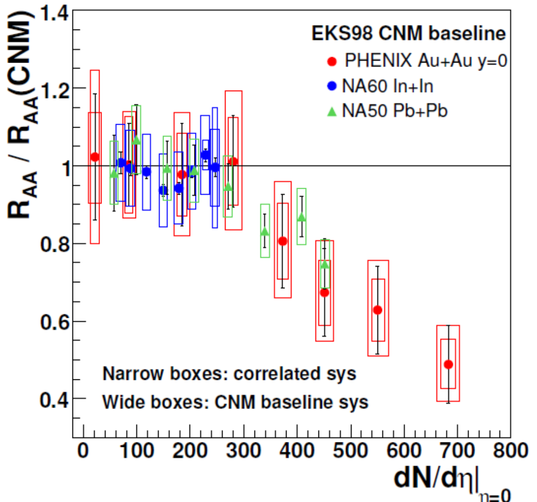
Plan of this talk

- Introduction
  - Motivation (QGP, quarkonia, heavy quark diffusion…)
  - Correlation and spectral functions
- Results
  - Quarkonium correlators and related quantities at $T > 0$
  - Quarkonium spectral functions at $T > 0$ (preliminary!)
- Conclusions and outlook

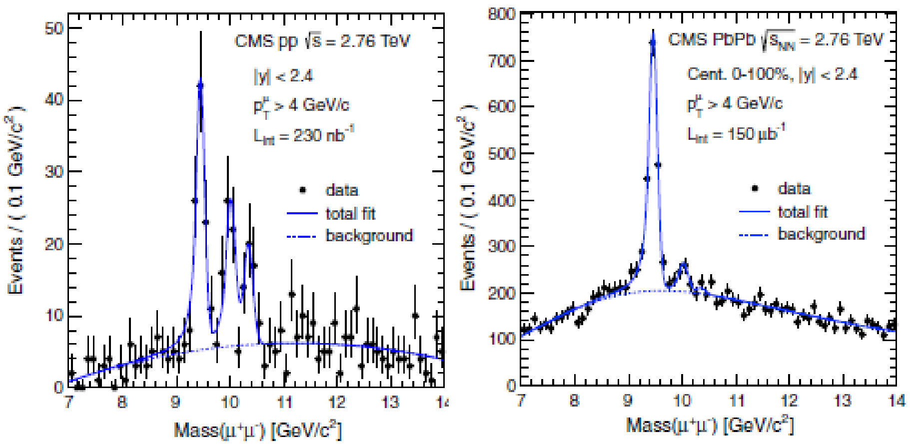
Quarkonia

- Bound states of heavy $q\bar{q}$
- At a certain temperature T_D, the dissociation should occur due to the color Debye screening
- An important probe of the quark-gluon plasma created in relativistic heavy ion collisions at RHIC, LHC

→ Theoretical investigation of in-medium properties of quarkonia plays an important role to understand experimental results.

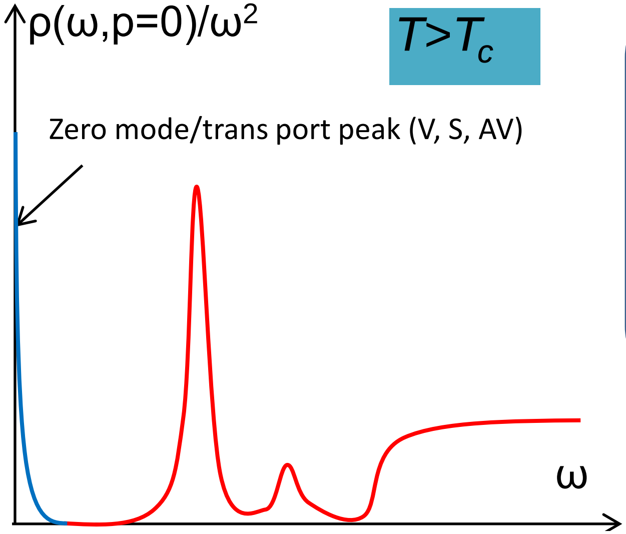


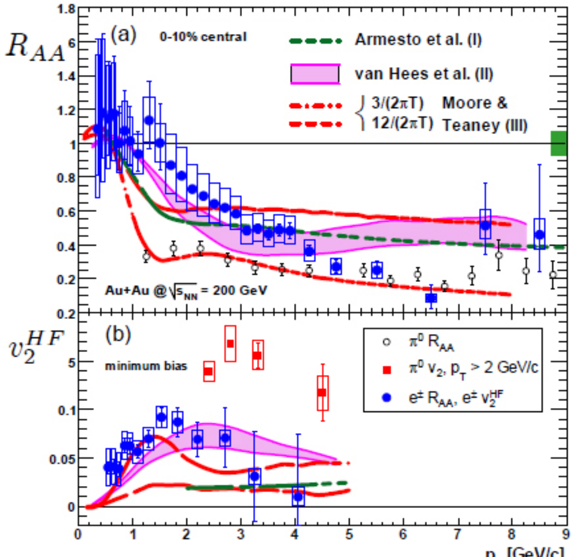
N. Brambilla *et al.*, EPJ C71 (2011) 1534



S. Chatrchyan *et al.*, PRL 109 (2012) 222301

Transport coefficients





Adare *et al*. [PHENIX Collaboration], PRL 98 (2007) 172301

Heavy quark diffusion coefficient

$$D = \frac{1}{6\chi_{00}} \lim_{\omega\to 0} \sum_{i=1}^{3} \frac{\rho_{ii}^V(\omega, \mathbf{0})}{\omega}$$

$\rho_{ii}^V(\omega)$: spatial component of vector spectral function

χ_{00} : Quark number susceptibility

$\rho_{00}^V(\omega) = 2\pi\chi_{00}\omega\delta(\omega)$ ➡ $G_{00}^V(\tau) = T\chi_{00}$

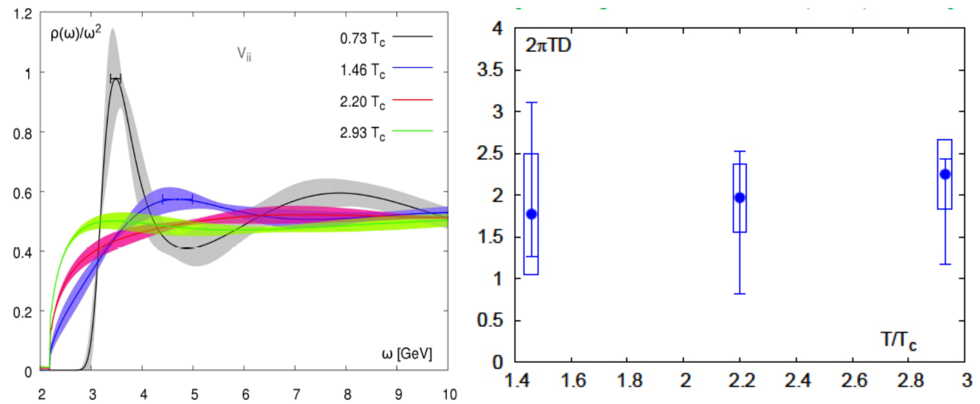
The evolution of the system in hydro models

→ **Transport coefficients are important.**

Determination by first principle (lattice QCD) calculations is needed.

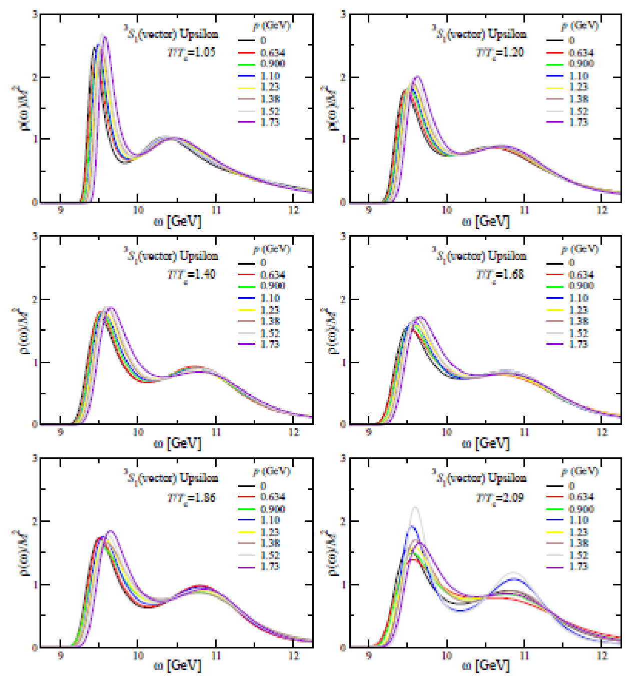
Recent lattice studies : spectral functions

- Charmonia
  - Several studies both in quenched QCD and with dynamical quarks
  - Dissociation temperatures are still not conclusive
  - A transport coefficient has been computed
  - **More precise determination of the SPFs on larger and finer lattice is needed**



H.-T. Ding *et al.*, PRD 86 (2012) 014509

- Bottomonia
  - NRQCD
  - **It is good to crosscheck without NRQCD**



G.Aarts *et al.*, PRL 106 (2011) 061602

G.Aarts *et al.*, JHEP 1303 (2012) 084

Quarkonium correlation and spectral functions

Euclidian (imaginary time) meson correlation function

$$G_H(\tau,\vec{p}) \equiv \int d^3x e^{-i\vec{p}\cdot\vec{x}} \langle J_H(\tau,\vec{x}) J_H(0,\vec{0}) \rangle$$

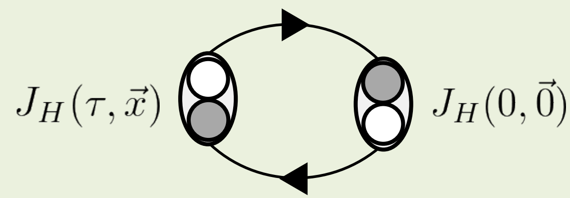
$$= \int_0^\infty \frac{d\omega}{2\pi} \rho_H(\omega,\vec{p}) K(\omega,\tau)$$

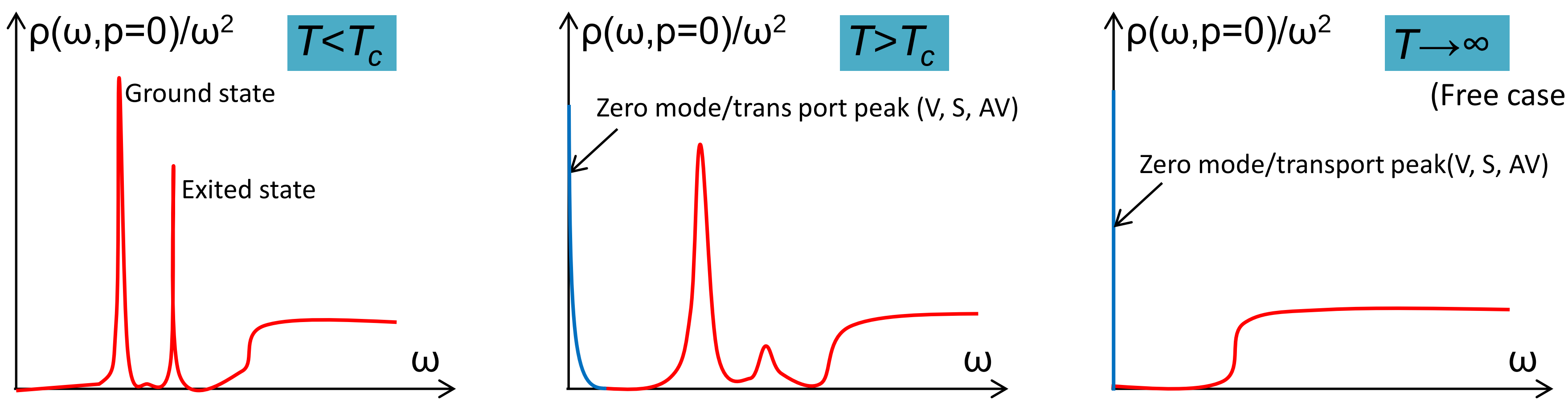
Spectral function (SPF)

has all information about in-medium meson properties

$$J_H(\tau,\vec{x}) \equiv \bar{\psi}(\tau,\vec{x}) \Gamma_H \psi(\tau,\vec{x})$$

$$K(\omega,\tau) \equiv \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}$$





Investigating spectral function

- Computing SPF → **Ill-posed problem**
  - # of data points of a correlator is $O(10)$ while a SPF needs O(1000) data points.
  - In general, simple χ^2 fitting does not work!
- Indirect investigation
  - Screening mass
  - Correlator itself
- Several ways to reconstruct SPF
  - Maximum entropy method (MEM) — M. Asakawa, T. Hatsuda and Y. Nakahara, Prog.Part.Nucl.Phys. 46 (2001) 459-508
  - A new Bayesian method — Y. Burnier and A. Rothkopf, PRL 111 (2013) 18, 182003
  - Stochastic methods

Larger number of data points is needed to reconstruct a SPF more precisely
→ Using large and fine lattice

Simulation setup

- Standard plaquette gauge & O(a)-improved Wilson quark actions
- In quenched QCD
- On fine and large isotropic lattices
- Two lattice spacings
- $T = 0.73 - 2.2T_c$
- Both charm & bottom

β	N_σ	N_τ	T/T_c	# confs.
7.192	96	48	0.73	259
		32	1.1	476
		28	1.25	336
		24	1.5	336
		16	2.2	239
7.793	192	96	0.73	69
		64	1.1	103
		56	1.25	190
		48	1.5	210

β	a [fm]	a^{-1} [GeV]	$\kappa_{\rm charm}$	$\kappa_{\rm bottom}$	$m_{J/\Psi}$ [GeV]	m_Υ [GeV]
7.192	0.0188	10.5	0.13194	0.12257	3.140(3)	9.574(3)
7.793	0.00942	20.9	0.13221	0.12798	3.175(5)	9.687(5)

The scale has been set by r_0=0.49fm and with a formula for r_0/a in

A. Francis, O. Kaczmarec, M. Laine, T. Neuhaus, HO, PRD 91 (2015) 9, 096002

Experimental values: $m_{J/\psi}$ = 3.096.916(11) GeV, m_Υ = 9.46030(26) GeV

J. Beringer *et al*. [PDG], PRD 86 (2012) 010001

Screening mass

Spatial meson correlation function

Screening mass

$$G(z) \equiv \int dxdy \int_0^{1/T} d\tau \langle J_H(\tau, \vec{x}) J_H(0, \vec{0}) \rangle \xrightarrow[z >> 1/T]{} e^{-M_{\mathrm{scr}} z}$$

If there is a lowest lying bound state

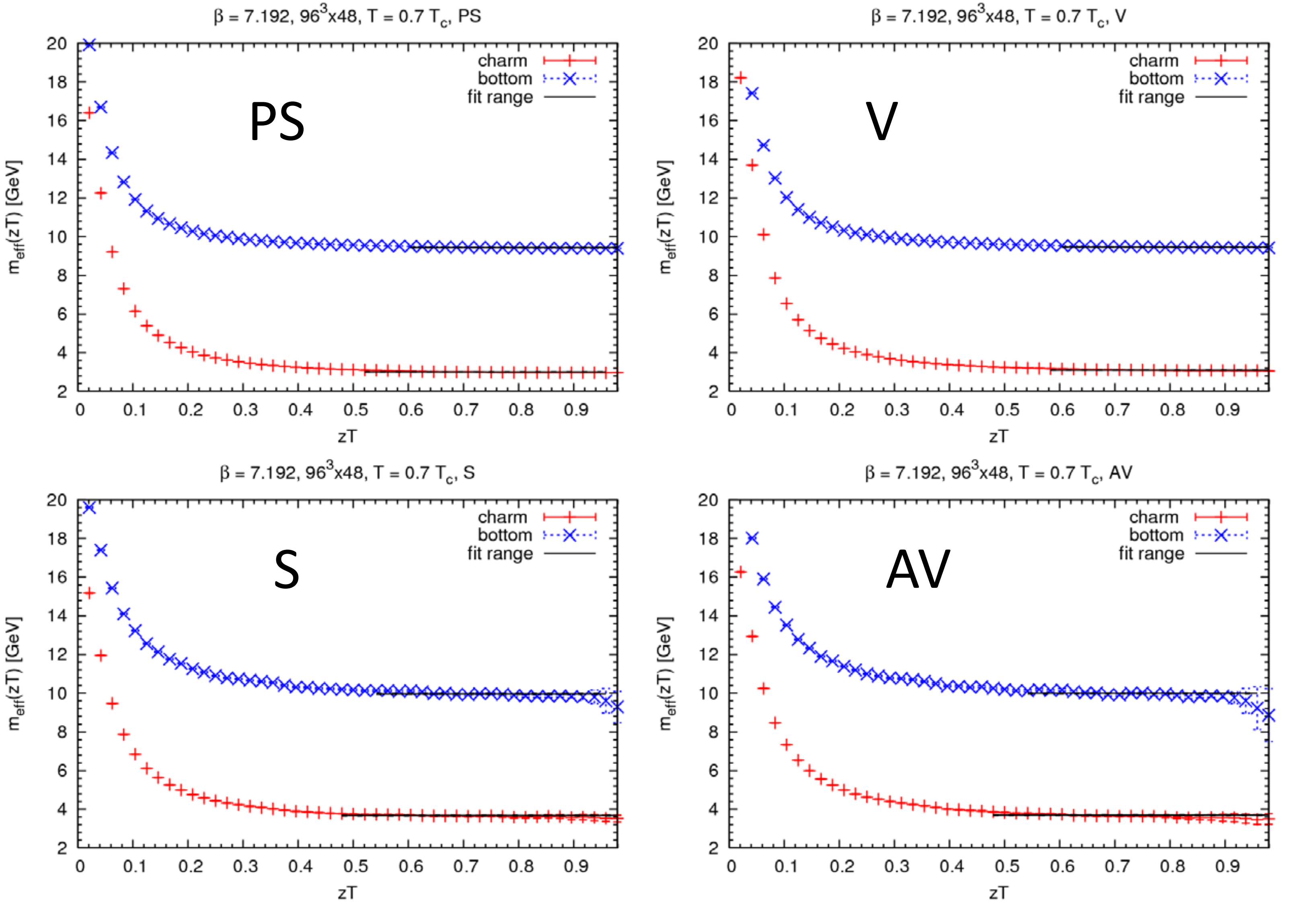
$$M_{\mathrm{scr}} = M$$

High *T* limit (free case)

Quark mass

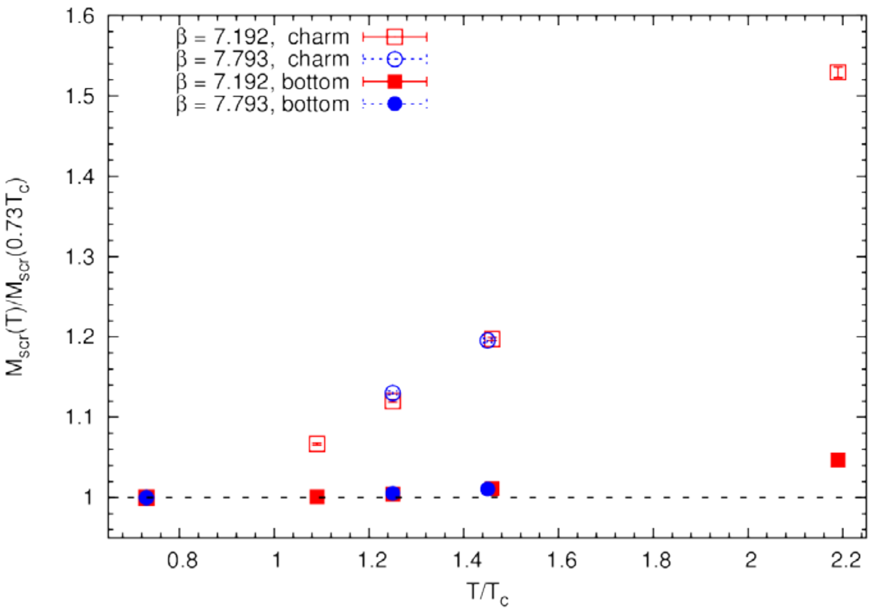
$$M_{\mathrm{scr}} = 2\sqrt{(\pi T)^2 + m_q^2}$$

Screening mass fits (β = 7.192, 0.7T_c)

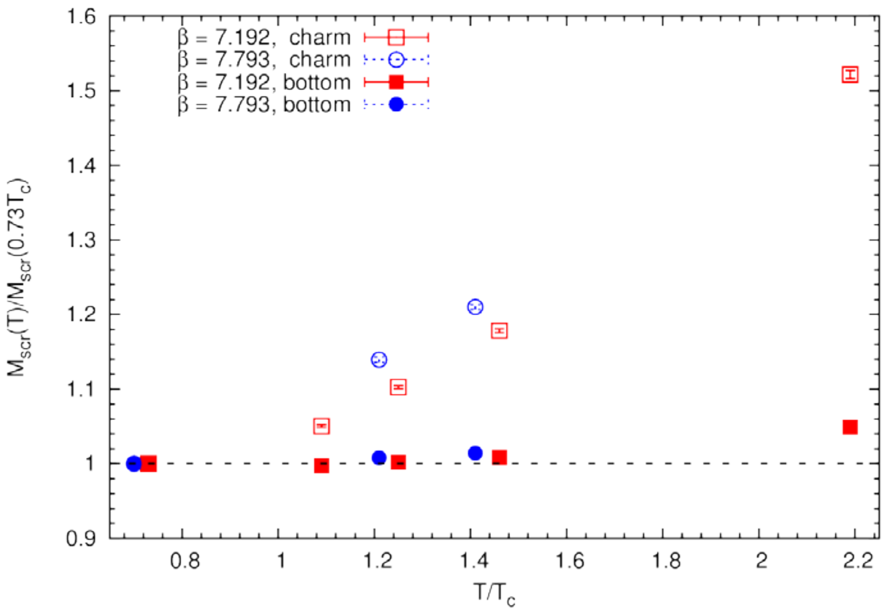


Screening mass for S-wave states

PS



V

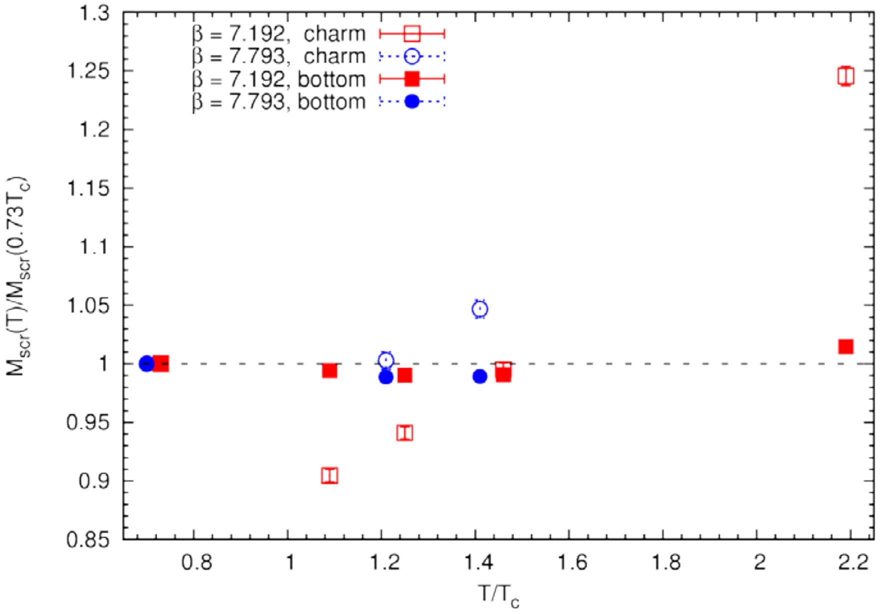


M_{scr} increases monotonically as increasing temperature.

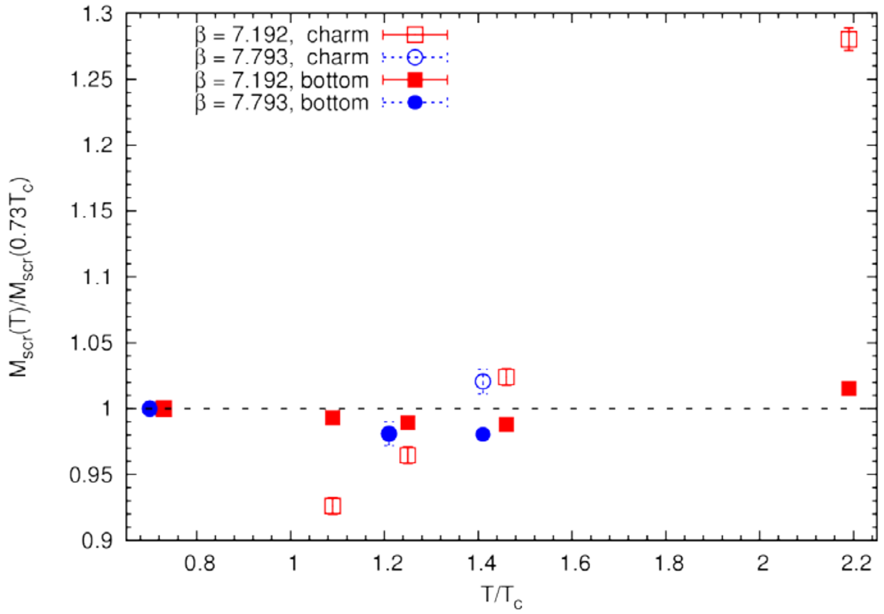
Small temperature dependence for bottom.

Screening mass for P-wave states

S



AV



M_{scr} has non-monotonic behavior.

Small temperature dependence for bottom.

Reconstructed correlator

$$G_{\rm rec}(\tau, T; T') \equiv \int_0^\infty d\omega \rho(\omega, T') K(\omega, \tau, T)$$

$$\frac{G(\tau, T)}{G_{\rm rec}(\tau, T; T')} = \frac{\int_0^\infty d\omega \rho(\omega, T) K(\omega, \tau, T)}{\int_0^\infty d\omega \rho(\omega, T') K(\omega, \tau, T)}$$

equals to unity at all τ

different — same

if the spectral function doesn't vary with temperature

S. Datta *et al.*, PRD 69 (2004) 094507

$$\frac{\cosh[\omega(\tau - N_\tau/2)]}{\sinh[\omega N_\tau/2]} = \sum_{\tau'=\tau;\Delta\tau'=N_\tau}^{N'_\tau - N_\tau + \tau} \frac{\cosh[\omega(\tau' - N'_\tau/2)]}{\sinh[\omega N'_\tau/2]}$$

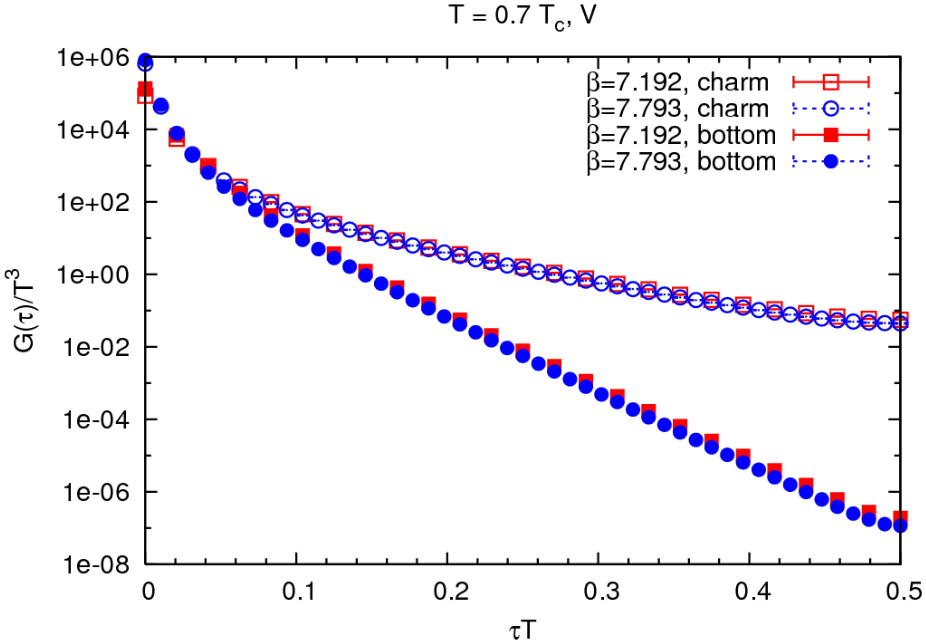
$T = 1/(N_\tau a) \qquad N'_\tau = m N_\tau \qquad m = 1, 2, 3, \cdots$

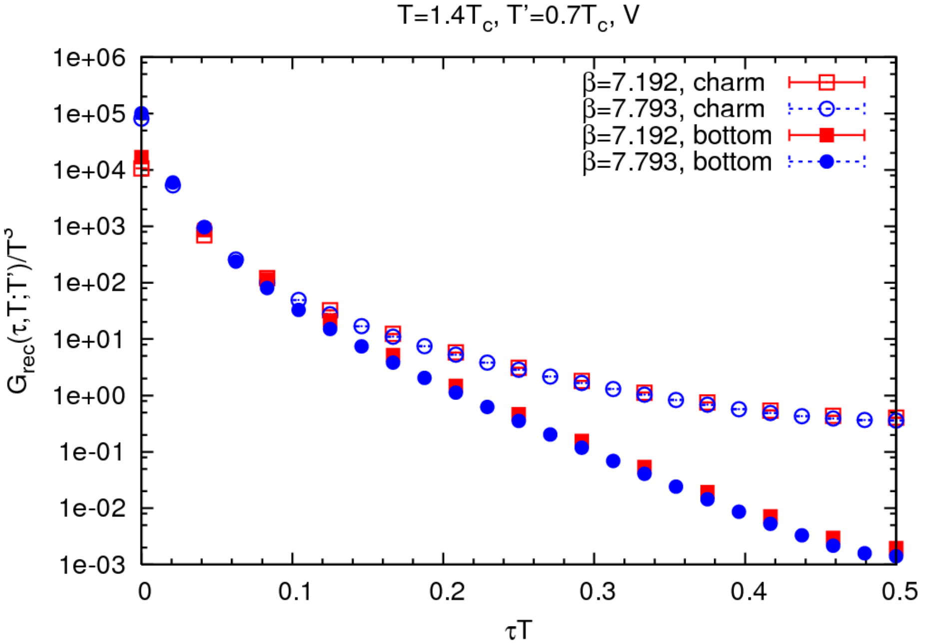
$$G_{\rm rec}(\tau, T; T') = \sum_{\tau'=\tilde{\tau};\Delta\tau'=N_\tau}^{N'_\tau - N_\tau + \tau} G(\tau', T')$$

H.-T. Ding *et al.*, PRD 86 (2012) 014509

Reconstructed correlator (2)

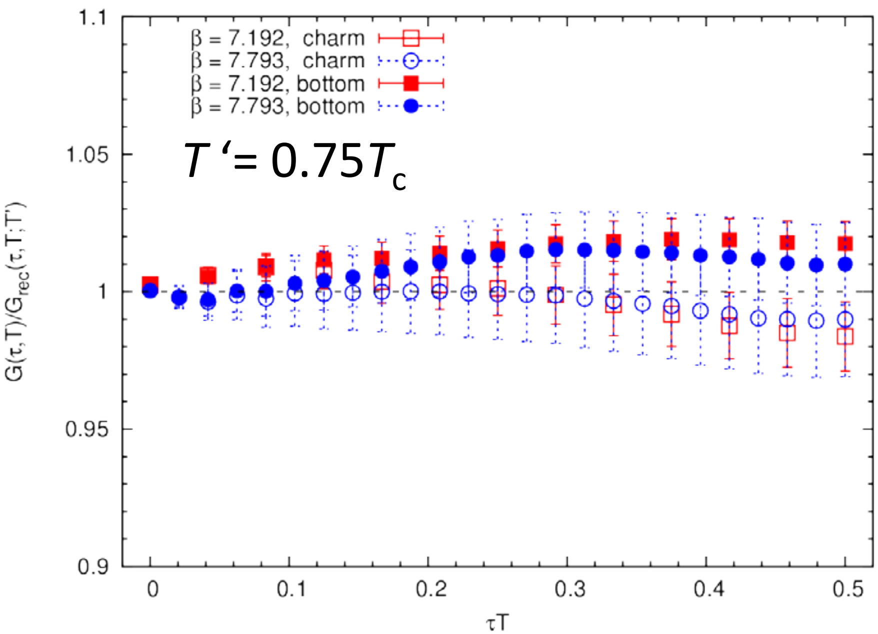
Ordinary correlator → Reconstructed correlator



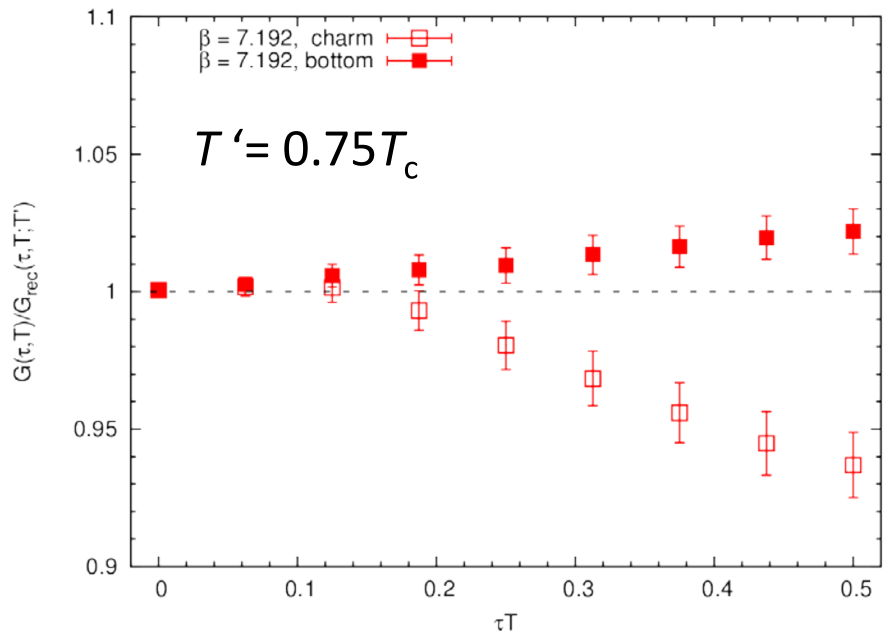


Reconstructed correlator for PS channel

$T = 1.5T_c$



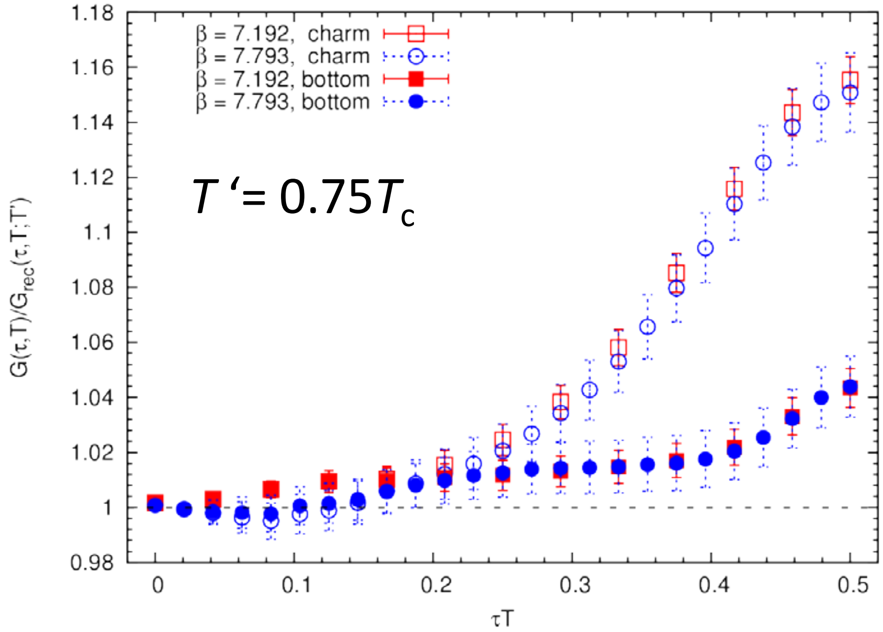
$T = 2.2T_c$



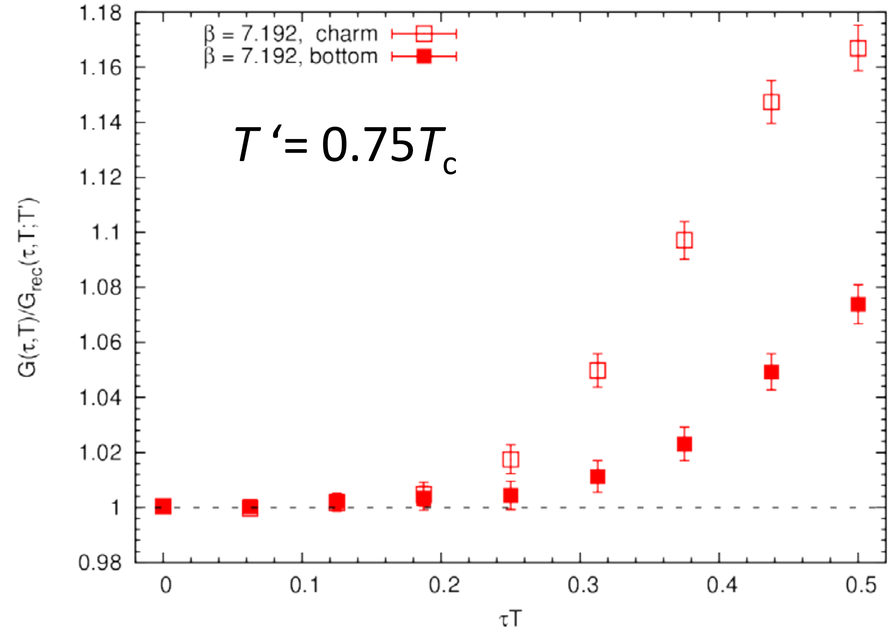
PS channel results have small thermal effect.
Charm has larger temperature dependence than bottom.

Reconstructed correlator for V channel

$T = 1.5T_c$



$T = 2.2T_c$

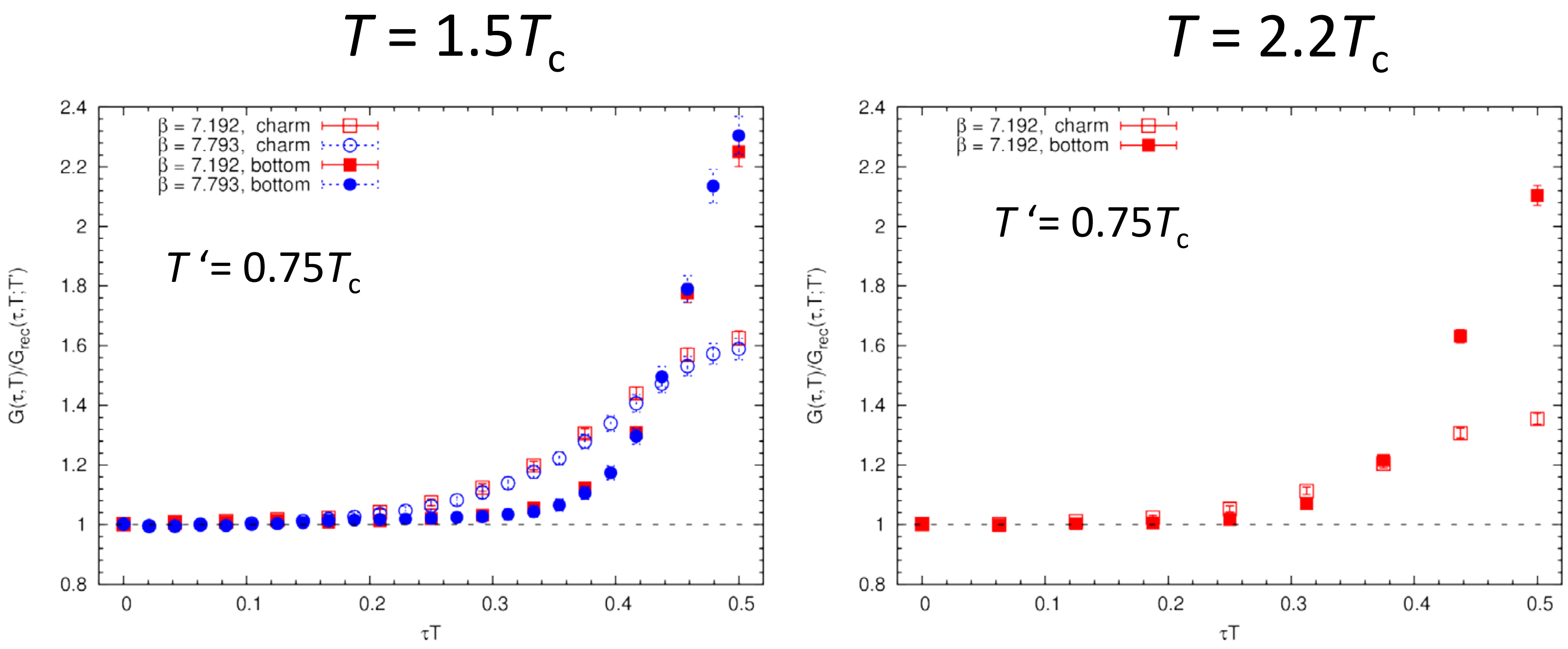


V channel results have strong enhancement at large τ, especially for charm.
Temperature dependence is small.

Large τ ↔ Small ω

→ This strong modification might be related to the transport peak.

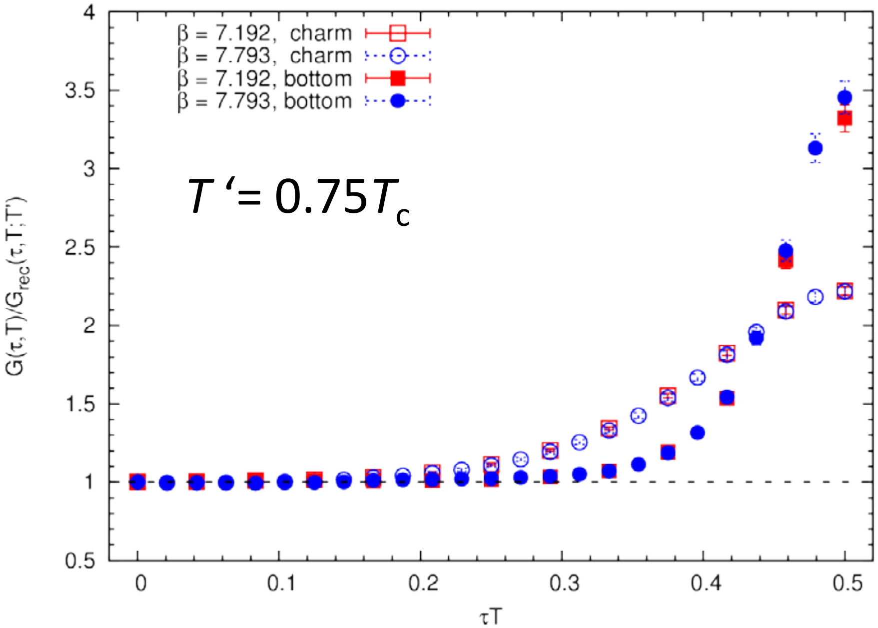
Reconstructed correlator for S channel

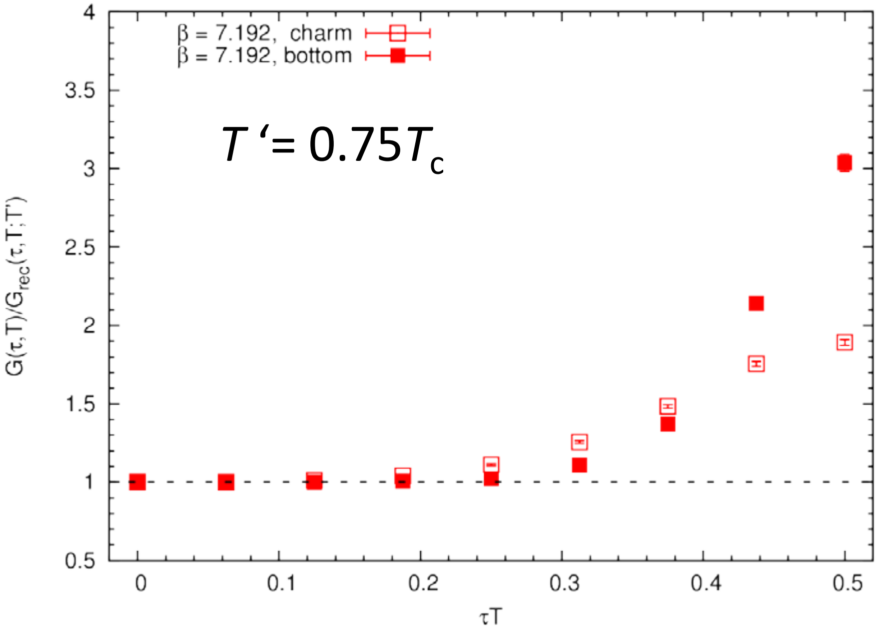


S channel results also have strong enhancement at large τ.
Temperature dependence is small.

Reconstructed correlator for AV channel

$T = 1.5T_c$ | $T = 2.2T_c$





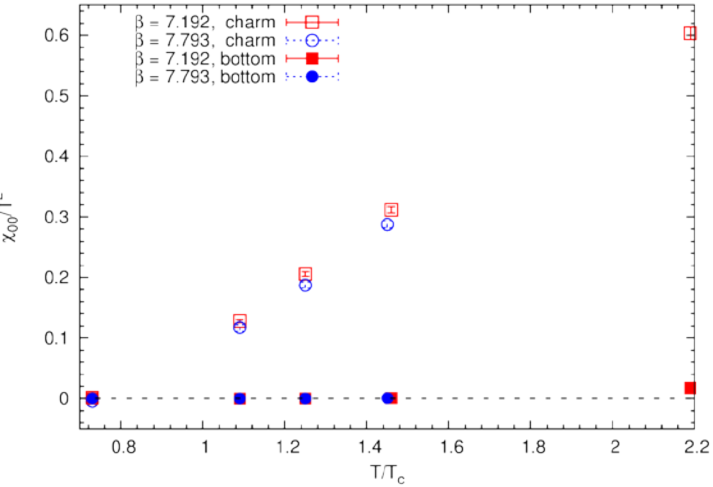
S channel results also have strong enhancement at large τ.
Temperature dependence is small.

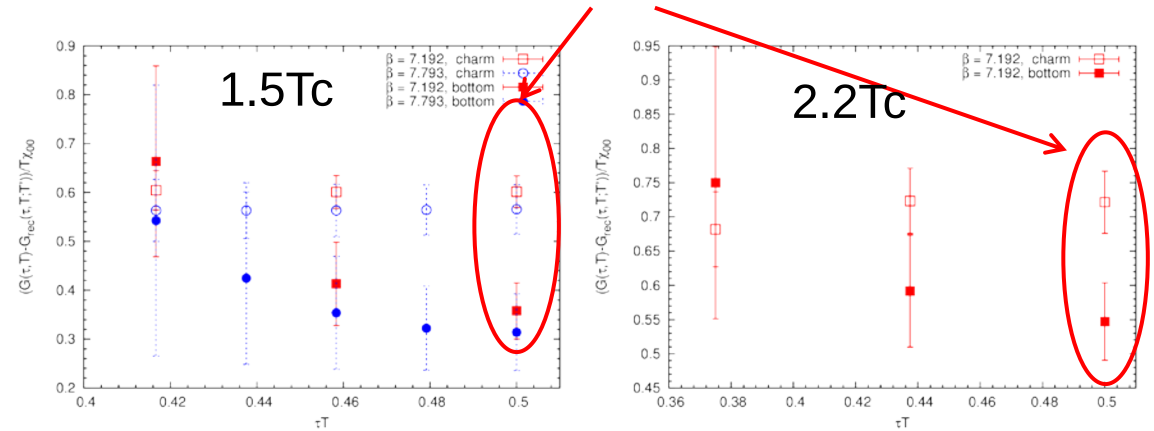
Estimating the heavy quark diffusion constant

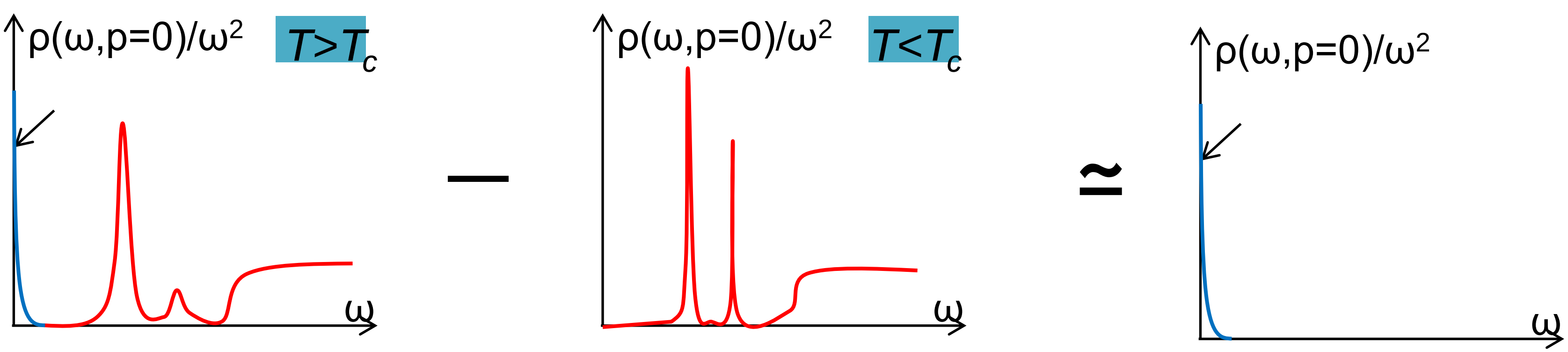
χ_{00} : Quark number susceptibility

$\rho^V_{00}(\omega) = 2\pi\chi_{00}\omega\delta(\omega)$ ➡ $G^V_{00}(\tau) = T\chi_{00}$

The contribution from the transport peak is assumed to be dominant in $G - G_{rec}$ at $\tau T = ½$.







Estimating the heavy quark diffusion constant (cont'd)

Heavy quark diffusion constant

$$D = \frac{1}{6\chi_{00}} \lim_{\omega \to 0} \sum_{i=1}^{3} \frac{\rho_{ii}^{V}(\omega, \mathbf{0})}{\omega}$$

Ansatz: $\rho_{ii}^{V}(\omega << T) = 2\chi_{00} \frac{T}{M} \frac{\omega\eta}{\omega^2 + \eta^2}$ $\quad \eta \equiv \frac{T}{MD}$ $\quad M \equiv m_q a$

H.-T. Ding *et al.*, PRD 86 (2012) 014509

m_q = 1—1.5 GeV

Charm: 2πTD ≈ 0.6—1 (β = 7.192), 2πTD ≈ 0.5—0.8 (β = 7.793) at 1.5Tc

2πTD ≈ 0.6—0.8 (β = 7.192) at 2.2Tc

Bottom: there is no solution for m_q > 4 GeV

Stochastic reconstruction of SPF

- Introducing a mapping $\phi : \mathbb{R} \mapsto [0,1]$

$$\phi(\omega) = \frac{1}{\mathcal{N}} \int_{-\infty}^{\omega} D(\nu) d\nu$$

Positive-definite
Same normalization to a spectral function

K.S.D. Beach
arXiv:cond-mat/0403055

- Regularization

$$A(\omega) \equiv \frac{1}{2\pi} \rho(\omega) \coth(\omega/2T) \quad \tilde{K}(\omega,\tau) \equiv K(\omega,\tau) \tanh(\omega/2T) = \frac{\cosh(\omega(\tau - 1/2T))}{\cosh(\omega/2T)}$$

- Normalization

$$G(0) = \int_0^{\infty} d\omega A(\omega) = \int d\phi(\omega) \frac{A(\omega)}{D(\omega)} = \int_0^1 dx n(x) \quad n(x) = \frac{A(\phi^{-1}(x))}{D(\phi^{-1}(x))}$$

- Averaging over all possible spectra weighted by $w \sim e^{-\chi^2/2\alpha}$

$$\langle n(x) \rangle = \frac{1}{Z} \int \mathcal{D}n \, n(x) e^{-\chi^2/2\alpha} \Rightarrow \langle A(\omega) \rangle = \langle n(\phi(\omega)) \rangle D(\omega)$$

$$\int \mathcal{D}n = \int_0^{\infty} \left(\prod_x dn(x) \right) \Theta(n) \delta \left(\int_0^1 dx n(x) - G(0) \right) \quad \chi^2 = \sum_{\tau,\tau'} \Delta(\tau) C^{-1}_{\tau,\tau'} \Delta(\tau')$$

$$\Delta(\tau) \equiv \int_0^1 dx n(x) \hat{K}(x,\tau) - G(\tau)$$

$$\hat{K}(\phi(\omega),\tau) = \tilde{K}(\omega,\tau)$$

Relation to MEM

- A mean-field treatment of the system of *n*(*x*) is equivalent to MEM

– Entropy

$$S = -\int_0^1 dx\bar{n}(x)\ln\bar{n}(x) = -\int d\omega\bar{A}(\omega)\ln\left(\frac{\bar{A}(\omega)}{D(\omega)}\right)$$

– Free energy

$$F\mathcal{N} = \frac{1}{2}\chi^2 - \alpha S - \mu\mathcal{N}$$

Default model
= prior information of SPF

MEM minimizes the free energy

Monte Carlo evaluation

1. Generating a configuration as superposition of δ functions

$$n_C(x) = \sum_\gamma r_\gamma \delta(x - a_\gamma)$$

– Update scheme:
  a. Shifting δ functions
  b. Changing residues of δ functions, keeping $\sum_\gamma r_\gamma = G(0)$
– Updating with probability $P = \min\{1, e^{-\Delta\chi^2/2\alpha}\}$

2. Taking ensemble average at a certain α $\langle n(x)\rangle_\alpha = \frac{1}{N}\sum_C n_c(x)$

3. Converting to the SPF $\langle A(\omega)\rangle_\alpha = D(\omega)\langle n(\phi(\omega))\rangle_\alpha$

Repeat 1-3 for various αs

Eliminating α

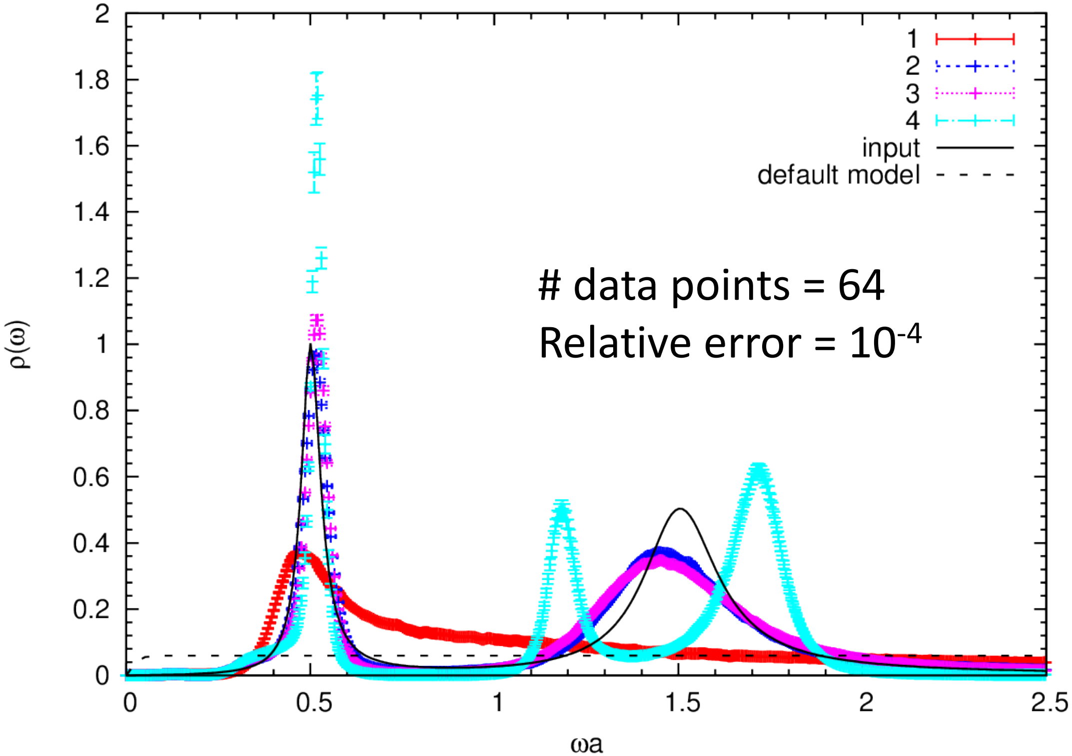
1. Plotting averaged entropy as a function of α and choosing α at a sharp drop in the entropy curve
   A. W. Sandvik, Phys. Rev. B 57, 10287 (1998)
2. Plotting χ^2 as a function of α and choosing α at a kink in the χ^2 curve
   K.S.D. Beach, arXiv:cond-mat/0403055
3. Simply choosing α at $\chi^2 \sim N_\tau$
4. Calculating a posterior probability $P[\alpha|G]$ from a Bayesian inference as MEM and choosing α which maximizes $P[\alpha|G]$ or averaging over all $\langle n(x) \rangle_\alpha$ weighted by $P[\alpha|G]$
   S. Fuchs *et al.*, PRE81, 056701 (2010)

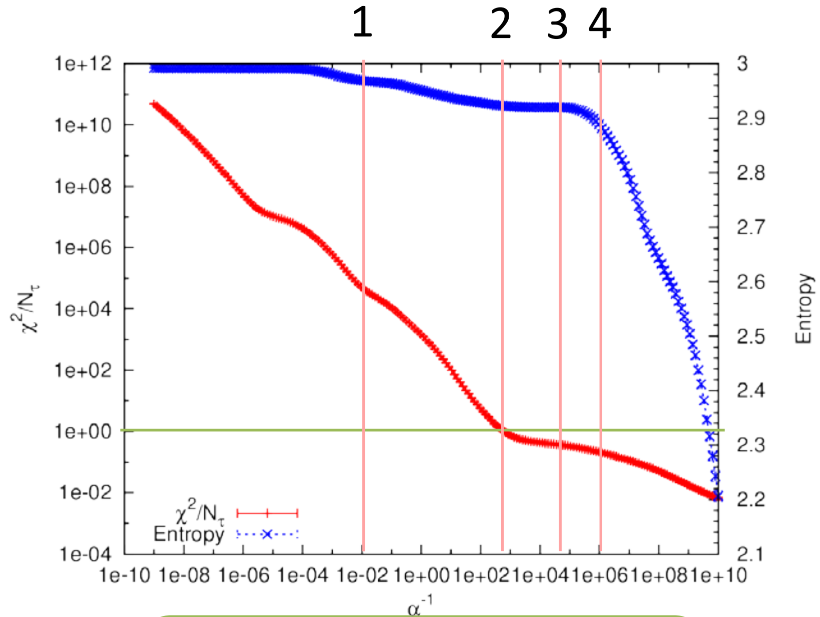
$$P[\alpha|G] \propto P[\alpha] \int \mathcal{D}n \; e^{-\chi^2/2\alpha} \qquad P[\alpha] = 1 \text{ or } 1/\alpha$$

Test with mock data

Averaged over 100 possible spectra

$D(\omega)$ = const.



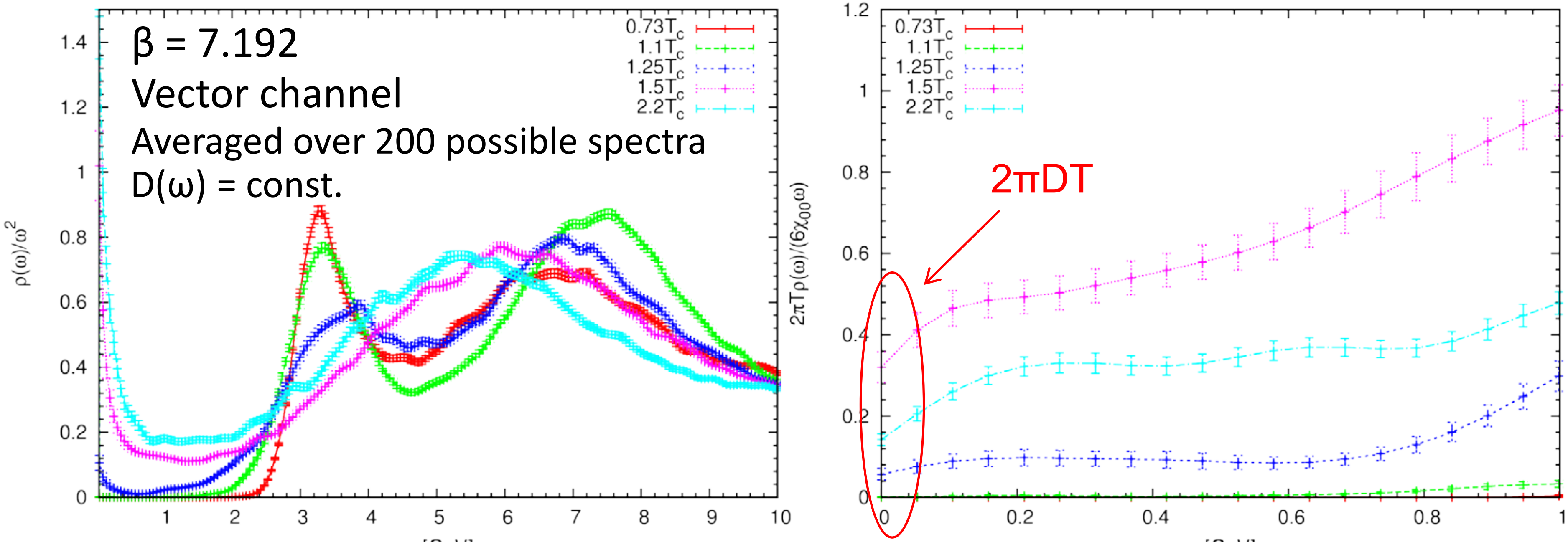


1: Very large α
2: $\chi^2/N_\tau = 1$
3: Before the entropy drop
4: After the entropy drop

α dependence is small in a range between 2 and 3.
The two-peak structure is reproduced well in this range.

Charmonium SPF at $T > 0$

Preliminary, only results for the V channel are shown



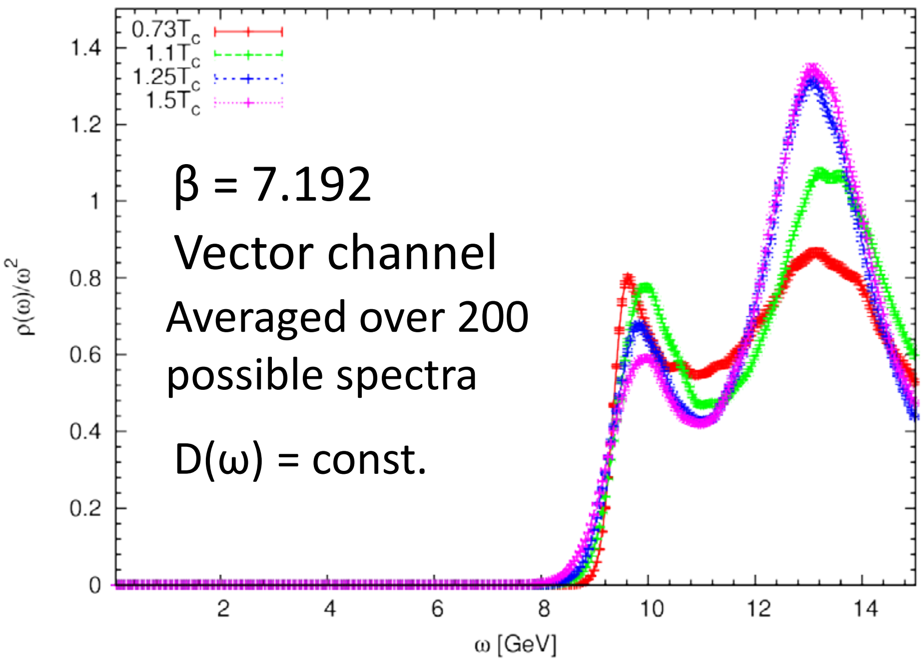
The J/Ψ peak might exist up to 1.25Tc.

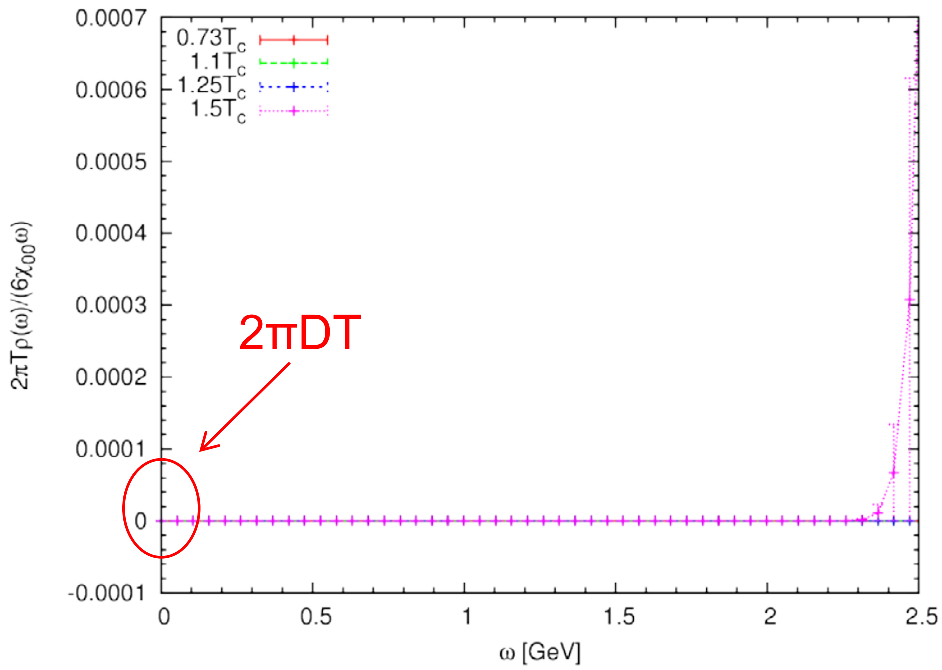
A transport peak appears at T > 1.25Tc.

2πDT has a slightly smaller value than what is estimated from the reconstructed correlator.

Bottomonium SPF at $T > 0$

Preliminary, only results for the V channel are shown





Data at 2.2Tc is not shown since it is unstable.

The Υ(1S) peak might exist up to 1.5Tc.

A transport peak is not visible up to 1.5Tc.

Summary

- We calculate meson correlation functions
  - on fine and large isotropic lattices
  - With 2 different cutoffs & quark masses for charm and bottom
- Screening masses
  - Different temperature dependence between S and P wave states
  - Small temperature dependence for bottomonia
- Meson spectral functions are investigated in terms of reconstructed correlators
  - V, S and AV channel have strong modification at large τ, which would be related to the transport peak.
- Charmonium and bottomonium spectral functions at finite temperature have been studied
  - with a stochastic method
  - J/Ψ seems to survive at $T < 1.25T_c$
  - A transport peak appears at $T > 1.25T_c$ for charm
  - Υ(1S) seems not to melt $T < 1.5T_c$
  - There is no clear signal of a transport peak for bottom even at $1.5T_c$
- The heavy quark diffusion constant is roughly estimated both form the reconstructed correlator and SPF, which is around 1 up to 2.2Tc for charm.

Outlook

- More studies for SPFs
  - Checking systematic uncertainty more carefully
    - Default model dependence, etc…
  - Eliminating α with the Bayesian inference
  - Comparison with MEM
- Estimating the heavy quark diffusion constant more precisely
- Finite momentum
- Taking continuum limit

End